Univerzitet u Zenici
Ekonomski fakultet
Zenica, 30.11.2010.

Parcijalni ispit iz predmeta Matematika

I grupa

1. Dokazati matematičkom indukcijom da važi:
$$\left(1-\frac{9}{2^2}\right)\left(1-\frac{9}{5^2}\right)\ldots\left[1-\frac{9}{(3n-1)^2}\right]=-\frac{3n+2}{2(3n-1)}\ (n\in\mathbb{N}).$$
2. Naći sve vrijednosti korjena $\sqrt[3]{z}$, ako je $z=\left(1-i\sqrt{3}\right)^5\left(\sqrt{3}+i\right)^{13}$.
3. Diskutovati rang matrice $M=\begin{bmatrix}1 & 2 & 1 & -1\\ -1 & -1 & 2 & 2\\ 2 & 3 & m+1 & m\\ 2 & 4 & 2m+1 & m+1\end{bmatrix}$ u zavisnosti od parametra.
4. Izračunati limese $L_1=\lim\limits_{n\to\infty}\dfrac{n\sqrt{1+3+5+\ldots+(2n+1)}}{2n^2+n+1}$ i $L_2=\lim\limits_{n\to\infty}\left(\dfrac{n^2}{n-5}-\dfrac{n^3+n^2+4n-1}{n^2-3n-10}\right)$.

II grupa

1. Izračunati x ako u binomnom razvoju $\left(\dfrac{\sqrt{2^x}}{\sqrt[16]{8}}+\dfrac{\sqrt[16]{32}}{\sqrt{2^x}}\right)^8$ dobijemo 56 kad oduzmemo šesti od četvrtog člana.
2. Riješiti jednačinu u skupu kompleksnih brojeva: $z^4-2z^2+9=0$.
3. Riješiti matričnu jednačinu $(3X)^{-1}+B^{-1}=(AX)^{-1}$, ako je
$$A=\begin{bmatrix}6 & 1 & -4\\ 1 & 3 & 2\\ 6 & 0 & -5\end{bmatrix},\ B=\begin{bmatrix}1 & -1 & 2\\ 1 & 2 & -1\\ -4 & 4 & 1\end{bmatrix}.$$
4. Izračunati limese $L_1=\lim\limits_{n\to\infty}\left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdots\sqrt[2^n]{2}\right)$, $L_2=\lim\limits_{n\to\infty}\dfrac{n^2+3n+4}{(n+1)+(n+2)+\ldots+2n}$.

III grupa

1. Dokazati matematičkom indukcijom da važi:
$$\frac{1\cdot 2}{3!}+\frac{2\cdot 2^2}{4!}+\frac{3\cdot 2^3}{5!}+...+\frac{n\cdot 2^n}{(n+2)!}=1-\frac{2^{n+1}}{(n+2)!}\ (n\in\mathbb{N}).$$

2. Napisati u trigonometrijskom obliku broj $z=\dfrac{a\sqrt{b}+ib\sqrt{a}}{b\sqrt{a}-ai\sqrt{b}}$, pri čemu su a i b pozitivni realni brojevi i zatim izračunati $\sqrt{z}$.

3. Riješiti matričnu jednačinu $(AX+A)^{-1}=BA$, ako je $A=\begin{bmatrix}2&2&3\\1&-1&0\\-1&2&1\end{bmatrix}, B=\begin{bmatrix}2&5&-3\\1&2&0\\1&3&-2\end{bmatrix}$.

4. Izračunati limese $L_1=\lim\limits_{n\to\infty}\dfrac{1+5+5^2+...+5^{n-1}}{25^n-1}$, $L_2=\lim\limits_{n\to\infty}\left(\dfrac{2}{n^2+1}+\dfrac{4}{n^2+1}+\dfrac{6}{n^2+1}+...+\dfrac{4n}{n^2+1}\right)$.

IV grupa

1. Izračunati x ako u binomnom razvoju $\left(\dfrac{\sqrt{2^{x-1}}}{\sqrt[3]{2}}+\sqrt[3]{4}\cdot\sqrt{2^x}\right)^6$ važi: $9T_3-T_5=240$.

2. Izračunati vrijednost determinante $D=\begin{vmatrix}1&1&\varepsilon\\1&1&\varepsilon^2\\\varepsilon^2&\varepsilon&1\end{vmatrix}$, ako je $\varepsilon=-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$.

3. Riješiti sistem jednačina i diskutovati rješenja sistema u zavisnosti od parametra:
$$(m+1)x-2y+(m+2)z=2m$$
$$-2x+my-2z=-2$$
$$(m-1)x-y+z=m-1.$$

4. Izračunati limese $L_1=\lim\limits_{n\to\infty}\left(n^2+\sqrt[3]{n^4-n^6}\right)$, $L_2=\lim\limits_{n\to\infty}\left(\dfrac{1+4+7+...+(3n-2)}{3n+1}-\dfrac{n}{2}\right)$.

Neki zadaci nisu detaljno raspisani.

I grupa Ekonomija

1. $n=1 \Rightarrow 1-\frac{9}{2^2} = -\frac{3+2}{2\cdot 2}$

$\frac{4-9}{4} = -\frac{5}{4} \Rightarrow -\frac{5}{4} = -\frac{5}{4}$

$P: \left(1-\frac{9}{2^2}\right)\left(1-\frac{9}{5^2}\right)\cdots\left[1-\frac{9}{(3k-1)^2}\right] = -\frac{3k+2}{2(3k-1)}$

$n=k+1 \Rightarrow \underbrace{\left(1-\frac{9}{2^2}\right)\cdots\left(1-\frac{9}{(3k-1)^2}\right)}_{\text{prema } P:\ -\frac{3k+2}{2(3k-1)}}\cdot\left(1-\frac{9}{(3k+2)^2}\right) = -\frac{3k+5}{2(3k+2)}$

$\Leftrightarrow -\frac{3k+2}{2(3k-1)}\cdot\frac{(3k+2)^2-3^2}{(3k+2)^2} = -\frac{3k+5}{2(3k+2)}$

$\Leftrightarrow -\frac{(3k+2-3)(3k+2+3)}{2(3k-1)(3k+2)} = -\frac{3k+5}{2(3k+2)}$

$\Leftrightarrow -\frac{3k+5}{2(3k+2)} = -\frac{3k+5}{2(3k+2)}$

2. $\begin{cases} 1-i\sqrt{3} = 2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right) \\ \sqrt{3}+i = 2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right) \end{cases}$

$\Rightarrow z = 2^5\left(\cos\frac{25\pi}{3}+i\sin\frac{25\pi}{3}\right)\cdot 2^{13}\cdot\left(\cos\frac{13\pi}{6}+i\sin\frac{13\pi}{6}\right)$

$z = 2^{18}\cdot\left[\cos\left(\frac{25\pi}{3}+\frac{13\pi}{6}\right)+i\sin\left(\frac{25\pi}{3}+\frac{13\pi}{6}\right)\right]$

$= 2^{18}\cdot\left(\cos\frac{50\pi+13\pi}{6}+i\sin\frac{50\pi+13\pi}{6}\right)$

$= 2^{18}\left(\cos\frac{63\pi}{6}+i\sin\frac{63\pi}{6}\right)$

$= 2^{18}\left(\cos\frac{21\pi}{2}+i\sin\frac{21\pi}{2}\right)$ / $\frac{21\pi}{2} = \underbrace{10\pi}_{\text{period}}+\frac{\pi}{2}$ /

$= 2^{18}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$$\sqrt[3]{z} = \underbrace{\sqrt[3]{2^{18}}}_{2^6}\left(\cos\frac{\frac{\pi}{2}+2k\pi}{3} + i\sin\frac{\frac{\pi}{2}+2k\pi}{3}\right), \quad k=0,1,2$$

$$k=0 \Rightarrow \sqrt[3]{z} = 64\cdot\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right) = 64\cdot\left(\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)$$
$$= 32(\sqrt{3}+i)$$

$$k=1 \Rightarrow \sqrt[3]{z} = 64\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$$
$$= 64\cdot\left(-\frac{\sqrt{3}}{2}+i\cdot\frac{1}{2}\right)$$
$$= 32(-\sqrt{3}+i)$$

$$k=2 \Rightarrow \sqrt[3]{z} = 64\left(\cos\frac{9\pi}{6}+i\sin\frac{9\pi}{6}\right) = 64\left(\underbrace{\cos\frac{3\pi}{2}}_{0}+i\underbrace{\sin\frac{3\pi}{2}}_{-i}\right)$$
$$= -64i$$

$$\sqrt[3]{z} \in \{32(\pm\sqrt{3}+i), -64i\}$$

3.

$$M = \begin{bmatrix} 1 & 2 & 1 & -1 \\ -1 & -1 & 2 & 2 \\ 2 & 3 & m+1 & m \\ 2 & 4 & 2m+1 & m+1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & -1 & m-1 & m+2 \\ 0 & 0 & 2m-1 & m+3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & m+2 & m+3 \\ 0 & 0 & 2m-1 & m+3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & m+2 & m+3 \\ 0 & 0 & m-3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & m-3 & 0 \\ 0 & 0 & m+2 & m+3 \end{bmatrix} \quad III\cdot(m+2)+IV\cdot(3-m)$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & m-3 & 0 \\ 0 & 0 & 0 & (m+3)\cdot(3-m) \end{bmatrix}$$

$m = 3 \Rightarrow r(M) = 2$

$m = -3 \Rightarrow r(M) = 3$

$m \neq \pm 3 \Rightarrow r(M) = 4$

4. $1+3+5+\ldots+2n+1 =$ /suma aritmetičkog niza/

$= (2\cdot 0+1)+(2\cdot 1+1)+\ldots+(2n+1) =$ /ima $(n+1)$ sabiraka/

$= \frac{n+1}{2}\cdot(1+2n+1) = \frac{n+1}{2}\cdot(2n+2) = \frac{n+1}{2}\cdot 2(n+1) = (n+1)^2$

$$\lim_{n\to\infty} \frac{n\sqrt{(n+1)^2}}{2n^2+n+1} = \lim_{n\to\infty} \frac{n(n+1)}{2n^2+n+1}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{2n^2+n+1} = \lim_{n\to\infty} \frac{n^2}{2n^2} = \frac{1}{2}.$$

$$L_2 = \lim_{n\to\infty}\left(\frac{n^2}{n-5} - \frac{n^3+n^2+4n-1}{n^2-3n-10}\right)$$

$$= \lim_{n\to\infty}\left(\frac{n^2}{n-5} - \frac{n^3+n^2+4n-1}{(n-5)(n+2)}\right)$$

$$= \lim_{n\to\infty} \frac{n^2(n+2)-(n^3+n^2+4n-1)}{(n-5)(n+2)}$$

$$= \lim_{n\to\infty} \frac{n^3+2n^2-n^3-n^2-4n+1}{n^2-3n-10} = \lim_{n\to\infty} \frac{n^2}{n^2} = 1$$

II grupa

1. $T_4 - T_6 = 56$

$T_4 = \binom{8}{3}\left(\frac{\sqrt{2^x}}{\sqrt[16]{8}}\right)^5 \cdot \left(\frac{\sqrt[16]{32}}{\sqrt{2^x}}\right)^3 =$

$= \frac{8\cdot7\cdot6}{1\cdot2\cdot3}\cdot\frac{2^{\frac{5x}{2}}}{2^{\frac{3\cdot5}{16}}}\cdot\frac{2^{\frac{5\cdot3}{16}}}{2^{\frac{3x}{2}}} = 56\cdot 2^{\frac{5x}{2}-\frac{3x}{2}} = 56\cdot2^x$

$T_6 = \binom{8}{5}\cdot\left(\frac{\sqrt{2^x}}{\sqrt[16]{8}}\right)^3\cdot\left(\frac{\sqrt[16]{32}}{\sqrt{2^x}}\right)^5 =$

$= 56\cdot\frac{2^{\frac{3x}{2}}}{2^{\frac{3\cdot3}{16}}}\cdot\frac{2^{\frac{5\cdot5}{16}}}{2^{\frac{5x}{2}}} = 56\cdot2^{\frac{3x}{2}-\frac{5x}{2}+\frac{25}{16}-\frac{9}{16}}$

$= 56\cdot2^{1-x}$

$56\cdot2^x - 56\cdot2^{1-x} = 56 \;/:56$

$2^x - 2^{1-x} = 1 \Rightarrow 2^x - \frac{2}{2^x} = 1 \;,\; 2^x = t$

$t - \frac{2}{t} = 1 \;/\cdot t \Rightarrow t^2 - t - 2 = 0$

$t_1 = 2,\quad t_2 = -1$

$2^x = 2 \Rightarrow \boxed{x_1 = 1}$

$2^x = -1 \Rightarrow$ nema rješenja

2. $z^4 - 2z^2 + 9 = 0$

$z^2 = t \Rightarrow t^2 - 2t + 9 = 0$

$D = 4 - 36 = -32$

$t_{1,2} = \frac{2\pm\sqrt{-32}}{2} = \frac{2\pm4\sqrt{2}\,i}{2} = 1\pm2\sqrt{2}\,i$

$z^2 = t \Rightarrow z = \sqrt{t}$

$$z_{1,2} = \sqrt{1+2\sqrt{2}i}$$

$$a=1,\ b=2\sqrt{2} \Rightarrow \sqrt{a^2+b^2} = \sqrt{1+8} = \sqrt{9} = 3$$

$$1+2\sqrt{2}i = 3\Big(\underbrace{\frac{1}{3}}_{\cos\varphi} + \underbrace{\frac{2\sqrt{2}}{3}}_{\sin\varphi} i\Big)$$

$$\sqrt{1+2\sqrt{2}i} = \sqrt{3}\cdot\left(\cos\frac{\varphi+2k\pi}{2} + i\sin\frac{\varphi+2k\pi}{2}\right),\ k=0,1$$

$$k=0 \Rightarrow z_1 = \sqrt{3}\cdot\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right)$$

$$\cos\frac{\varphi}{2} = \sqrt{\frac{1+\cos\varphi}{2}} = \sqrt{\frac{1+\frac{1}{3}}{2}} = \sqrt{\frac{\frac{4}{3}}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin\frac{\varphi}{2} = \sqrt{\frac{1-\cos\varphi}{2}} = \sqrt{\frac{1-\frac{1}{3}}{2}} = \sqrt{\frac{\frac{2}{3}}{2}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

$$z_1 = \sqrt{3}\cdot\left(\frac{\sqrt{2}}{\sqrt{3}} + i\cdot\frac{1}{\sqrt{3}}\right) = \sqrt{2}+i$$

$$z_2 = \sqrt{3}\cdot\left[\cos\left(\frac{\varphi}{2}+\pi\right) + i\sin\left(\frac{\varphi}{2}+\pi\right)\right]$$

$$= \sqrt{3}\cdot\left(-\cos\frac{\varphi}{2} - i\sin\frac{\varphi}{2}\right)$$

$$= \sqrt{3}\cdot\left(-\frac{\sqrt{2}}{\sqrt{3}} - i\cdot\frac{1}{\sqrt{3}}\right) = -\sqrt{2}-i$$

$$z_{1,2} = \pm(\sqrt{2}+i)$$

Analogno, $z_{3,4} = \pm(\sqrt{2}-i)$

3. $$(3X)^{-1} + B^{-1} = (AX)^{-1}$$

$$\frac{1}{3}X^{-1} + B^{-1} = X^{-1}A^{-1}$$

$$\frac{1}{3}X^{-1} - X^{-1}A^{-1} = -B^{-1}$$

$$X^{-1}\left(\frac{1}{3}I - A^{-1}\right) = -B^{-1} \quad /\cdot X \text{ lijevo}$$

$$\frac{1}{3}I - A^{-1} = -X\cdot B^{-1} \quad /\cdot(-B) \text{ desno}$$

$$X = \left(\tfrac{1}{3}I - A^{-1}\right)\cdot(-B)$$

$$\det|A| = \begin{vmatrix} 6 & 1 & -4 \\ 1 & 3 & 2 \\ 6 & 0 & -5 \end{vmatrix} = \begin{vmatrix} 6 & 1 & -4 \\ -17 & 0 & 14 \\ 6 & 0 & -5 \end{vmatrix} =$$

$$= -(85 - 84) = -1$$

$A_{11} = -15,\quad A_{12} = 17,\quad A_{13} = -18$

$A_{21} = 5,\quad A_{22} = -6,\quad A_{23} = +6$

$A_{31} = 14,\quad A_{32} = -16,\quad A_{33} = 17$

$$\Rightarrow A^{*} = \begin{bmatrix} -15 & 5 & 14 \\ 17 & -6 & -16 \\ -18 & 6 & 17 \end{bmatrix}$$

$$A^{-1} = -A^{*}$$

$$X = \left(\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} + \begin{bmatrix} -15 & 5 & 14 \\ 17 & -6 & -16 \\ -18 & 6 & 17 \end{bmatrix} \right) \cdot \begin{bmatrix} -1 & 1 & -2 \\ -1 & -2 & 1 \\ 4 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{44}{3} & 5 & 14 \\ 17 & -\frac{17}{3} & -16 \\ -18 & 6 & \frac{52}{3} \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 & -2 \\ -1 & -2 & 1 \\ 4 & -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{197}{3} & -\frac{242}{3} & \frac{59}{3} \\ -\frac{226}{3} & \frac{277}{3} & -\frac{71}{3} \\ \frac{244}{3} & -\frac{298}{3} & \frac{74}{3} \end{bmatrix}$$

4. $L_1 = \lim_{n\to\infty} \left(\sqrt{2} \cdot \sqrt[4]{2} \cdots \sqrt[2^n]{2}\right) = \lim_{n\to\infty} 2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} \cdots 2^{\frac{1}{2^n}} =$

$= \lim_{n\to\infty} 2^{\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}} = \lim_{n\to\infty} 2^{\frac{1}{2}\cdot\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}}$; $\left(\frac{1}{2}\right)^n \to 0$

$= 2.$

$L_2 = \lim_{n\to\infty} \frac{n^2+3n+4}{(n+1)+(n+2)+\cdots+2n} =$

$= \lim_{n\to\infty} \frac{n^2+3n+4}{\frac{n}{2}(n+1+2n)} = \lim_{n\to\infty} \frac{2(n^2+3n+4)}{3n^2+n} = \frac{2}{3}.$

III grupa E.F.

1. $n=1 \Rightarrow \frac{1\cdot 2}{3!} = 1 - \frac{2^2}{3!} \Rightarrow \frac{2}{6} = 1 - \frac{4}{6} \Rightarrow \frac{1}{3} = \frac{1}{3}$ ✓

P: $n=k \Rightarrow \frac{1\cdot 2}{3!} + \dots + \frac{k\cdot 2^k}{(k+2)!} = 1 - \frac{2^{k+1}}{(k+2)!}$

$n=k+1 \Rightarrow \underbrace{\frac{1\cdot 2}{3!} + \dots + \frac{k\cdot 2^k}{(k+2)!}}_{1 - \frac{2^{k+1}}{(k+2)!}} + \frac{(k+1)\cdot 2^{k+1}}{(k+3)!} = 1 - \frac{2^{k+2}}{(k+3)!}$

$\Leftrightarrow 1 - \frac{2^{k+1}}{(k+2)!} + \frac{(k+1)\cdot 2^{k+1}}{(k+3)!} = 1 - \frac{2^{k+2}}{(k+3)!}$

$1 + \frac{(k+1)\cdot 2^{k+1} - (k+3)\cdot 2^{k+1}}{(k+3)!} = 1 - \frac{2^{k+2}}{(k+3)!}$

$1 + \frac{2^{k+1}(k+1-k-3)}{(k+3)!} = 1 - \frac{2^{k+2}}{(k+3)!}$

$1 - \frac{2^{k+2}}{(k+3)!} = 1 - \frac{2^{k+2}}{(k+3)!}$

2. $z = \frac{a\sqrt{b} + ib\sqrt{a}}{b\sqrt{a} - ia\sqrt{b}} \cdot \frac{i}{i} = \frac{i(a\sqrt{b} + ib\sqrt{a})}{b\sqrt{a}\,i + a\sqrt{b}} = i$

$\sqrt{z} = \sqrt{i} = \sqrt{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}} = \cos\frac{\frac{\pi}{2}+2k\pi}{2} + i\sin\frac{\frac{\pi}{2}+2k\pi}{2}$ $(k=0,1)$

$k=0 \Rightarrow \sqrt{z} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$k=1 \Rightarrow \sqrt{z} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$\sqrt{z} = \pm\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \pm\frac{\sqrt{2}}{2}(1+i)$.

3. $(AX+A)^{-1}=BA \Rightarrow AX+A=A^{-1}B^{-1}$

$A\cdot X=A^{-1}B^{-1}-A \quad / \cdot A^{-1}$ lijevo

$X=A^{-1}(A^{-1}B^{-1}-A)$

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 3 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 4 & 3 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 4 & 3 & 1 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 & -6 & -4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 6 & 4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & -3 \\ 0 & 1 & 0 & 1 & -5 & -3 \\ 0 & 0 & 1 & -1 & 6 & 4 \end{array}\right], \quad A^{-1}=\begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 5 & -3 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 5 & -3 & 1 & 0 & 0 \\ 1 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 & -1 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & -2 & 5 & 0 \\ 0 & 1 & -3 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -1 & -6 \\ 0 & 1 & 0 & -2 & 1 & 3 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right], \quad B^{-1}=\begin{bmatrix} 4 & -1 & -6 \\ -2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 & -3 \\ 1 & 2 & 0 \\ 1 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 & -6 \\ -2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 & -6 \\ -2 & 1 & 3 \\ -1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 15 & -8 & -21 \\ 17 & -9 & -24 \\ -20 & 11 & 28 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 13 & -10 & -24 \\ 16 & -8 & -24 \\ -19 & 9 & 27 \end{bmatrix} = \begin{bmatrix} 6 & -5 & -9 \\ -10 & 3 & 15 \\ 7 & -2 & -12 \end{bmatrix}$$

4. $$L_1 = \lim_{n\to\infty} \frac{\frac{5^n-1}{5-1}}{(5^n-1)(5^n+1)} = \lim_{n\to\infty} \frac{1}{4(5^n+1)} = 0$$

$$L_2 = \lim_{n\to\infty} \frac{2+4+6+\dots+4n}{n^2+1} = \lim_{n\to\infty} \frac{2(1+2+\dots+2n)}{n^2+1}$$

$$= \lim_{n\to\infty} \frac{2 \cdot \frac{2n}{2}(1+2n)}{n^2+1} = \lim_{n\to\infty} \frac{2n+4n^2}{n^2+1} = 4$$

IV група Е.Г

1. $T_3 = \binom{6}{2}\left(\frac{\sqrt{2^{x-1}}}{\sqrt[3]{2}}\right)^4 \cdot \left(\sqrt[3]{4}\cdot\sqrt{2^x}\right)^2$

$= \frac{6\cdot 5}{1\cdot 2}\cdot \frac{2^{2(x-1)}}{\sqrt[3]{2^4}} \cdot \sqrt[3]{4^2}\cdot\left(\sqrt{2^x}\right)^2$

$= 15\cdot 2^{2x-2+x} = 15\cdot 2^{3x-2}$

$T_5 = \binom{6}{4}\left(\frac{\sqrt{2^{x-1}}}{\sqrt[3]{2}}\right)^2 \cdot \left(\sqrt[3]{4}\cdot\sqrt{2^x}\right)^4$

$= 15\cdot \frac{2^{x-1}}{\sqrt[3]{2^2}}\cdot \sqrt[3]{4^4}\cdot 2^{2x}$ ($4^4 = 2^8$)

$= 15\cdot 2^{3x-1}\cdot 2^{\frac{8}{3}-\frac{2}{3}} = 15\cdot 2^{3x-1}\cdot 2^2 = 60\cdot 2^{3x-1}$

$9T_3 - T_5 = 240$

$9\cdot 15\cdot 2^{3x-2} - 60\cdot 2^{3x-1} = 240$

$2^{3x-2}(135-120) = 240 \quad /:15$

$2^{3x-2} = 2^4$

$3x-2 = 4 \Rightarrow 3x = 6 \Rightarrow \boxed{x=2}$

2. $D = \begin{vmatrix} 1 & 1 & \varepsilon \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} \overset{I-II}{=} \begin{vmatrix} 0 & 0 & \varepsilon-\varepsilon^2 \\ 1 & 1 & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{vmatrix} =$

$= (\varepsilon-\varepsilon^2)\cdot(\varepsilon-\varepsilon^2) = [\varepsilon(1-\varepsilon)]^2 = \varepsilon^2(1-\varepsilon)^2$

$= \varepsilon^2(1-2\varepsilon+\varepsilon^2)$

$= \varepsilon^2 - 2\varepsilon^3 + \varepsilon^4$

$\varepsilon = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$\Rightarrow \varepsilon^3 = 1$

$D = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} - 2 + \cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}$, $\frac{8\pi}{3} = \underbrace{2\pi}_{\text{period}} + \frac{2\pi}{3}$

$D = -\cos\frac{\pi}{3} ~~+ i\sin\frac{\pi}{3}~~ - 2 - \cos\frac{\pi}{3} ~~+ i\sin\frac{\pi}{3}~~$

$D = -\frac{1}{2} - 2 - \frac{1}{2} = -3$

3. $D = (1-m)(m-2)(m+3)$

$D_x = (1-m)(m-2)(m+2)$

$D_y = (1-m)\cdot 2(m-2)$

$D_z = (1-m)(m-2)(m+1)$

1° $m \neq 1,\ m \neq 2,\ m \neq -3$ sistem ima jedno jedino rješenje

$\left(\frac{m+2}{m+3}, \frac{2}{m+3}, \frac{m+1}{m+3}\right)$

2° $m = 1 \Rightarrow D = D_x = D_y = D_z = 0$

$2x - 2y + 3z = 2$ ...(1)

$-2x + y - 2z = -2$ ...(2)

$-y + z = 0$ ...(3)

(1) + (2): $-y + z = 0 \Rightarrow y = z \Rightarrow 2x = 2 - y$

Rješenje: $\left(\frac{2-y}{2}, y, y\right)$, $y \in \mathbb{R}$

3° $m = -2 \Rightarrow D = D_x = D_y = D_z = 0$

$3x - 2y + 4z = 4$

$-2x + 2y - 2z = -2 \Rightarrow x - y + z = 1$

iste < $x - y + z = 1$

$z = 1 - x + y$

$3x-2y+4(1-x+y)=4$

$3x-2y+4-4x+4y=4$

$-x+2y=0 \Rightarrow \boxed{x=2y} \Rightarrow z=1-2y+y=1-y$

Рјешења: $(2y, y, 1-y),\ y\in R$

$5^\circ\ m=-3 \Rightarrow$ нема рјешења $(D=0, D_x\neq 0)$

4. $L_1=\lim_{n\to\infty}\left(n^2+\sqrt[3]{n^4-n^6}\right)\cdot\frac{n^4+n^2\sqrt[3]{n^4-n^6}+\left(\sqrt[3]{n^4-n^6}\right)^2}{n^4+n^2\cdot\sqrt[3]{n^4-n^6}+\sqrt[3]{(n^4-n^6)^2}}$

$=\lim_{n\to\infty}\frac{n^6+n^4-n^6}{n^4+n^2\cdot\sqrt[3]{n^4-n^6}+\sqrt[3]{(n^4-n^6)^2}}$

$=\lim_{n\to\infty}\frac{n^4}{n^4-n^4+n^4}=1$

$L_2=\lim_{n\to\infty}\left(\frac{1+4+7+\dots+3n-2}{3n+1}-\frac{n}{2}\right)$

$=\lim_{n\to\infty}\left(\frac{\frac{n}{2}(1+3n-2)}{3n+1}-\frac{n}{2}\right)$

$=\lim_{n\to\infty}\frac{n}{2}\left(\frac{3n-1}{3n+1}-1\right)$

$=\lim_{n\to\infty}\frac{n}{2}\cdot\frac{3n-1-3n-1}{3n+1}$

$=\lim_{n\to\infty}\frac{-2n}{6n+2}=-\frac{2}{6}=-\frac{1}{3}.$